Soft geometric inductive bias

for object centric dynamics

Hampus Linander 17/8 - 2026 GeUmetric workshop Umeå

Preprint

Soft Geometric Inductive Bias for Object Centric Dynamics

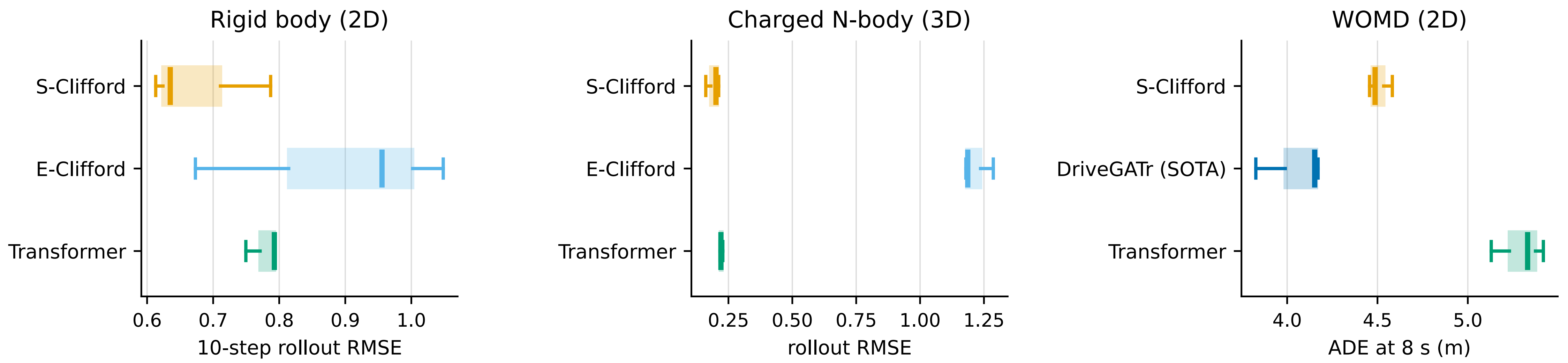
Hampus Linander[1] **Conor Heins**[2] **Alexander Tschantz**[3]
Marco Perin[4] **Christopher L Buckley**[3]

[1]Department of Mathematical Sciences
Chalmers University of Technology
University of Gothenburg

[2]Department of Collective Behavior
Max Planck Institute of Animal Behavior

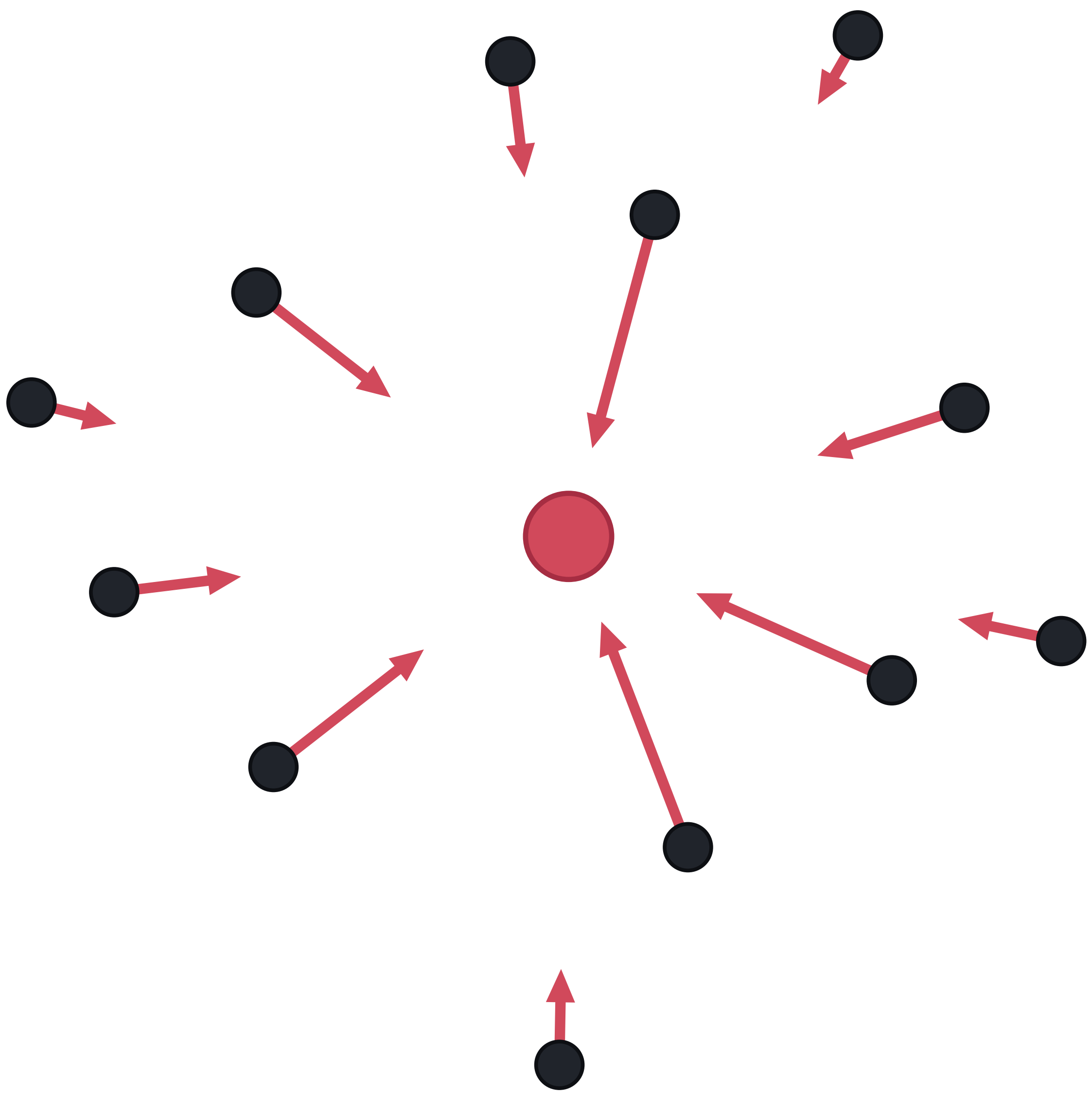
[3]Department of Informatics
University of Sussex

[4]Kodamai



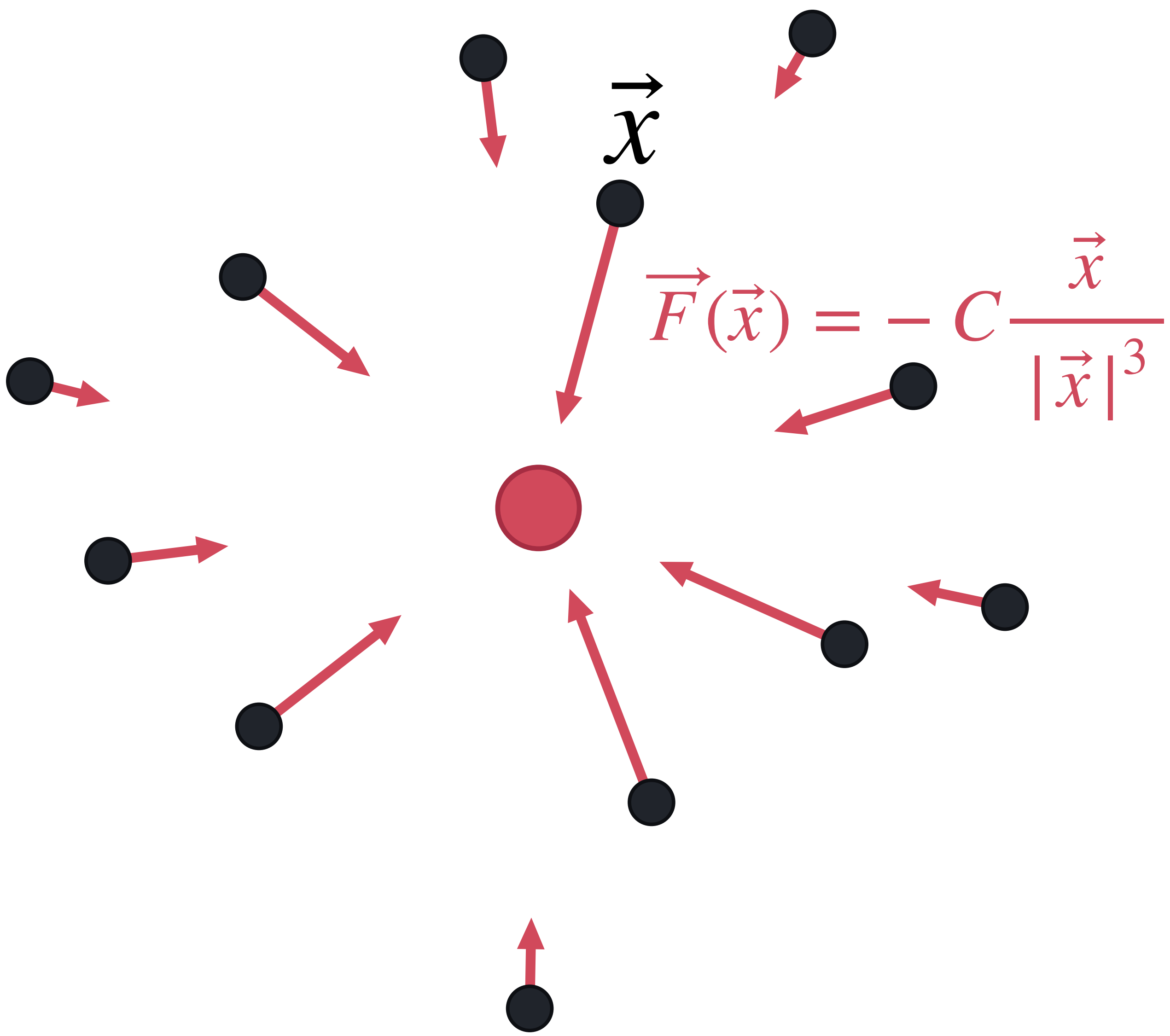
Inductive bias

Example: learn force field between two charged particles



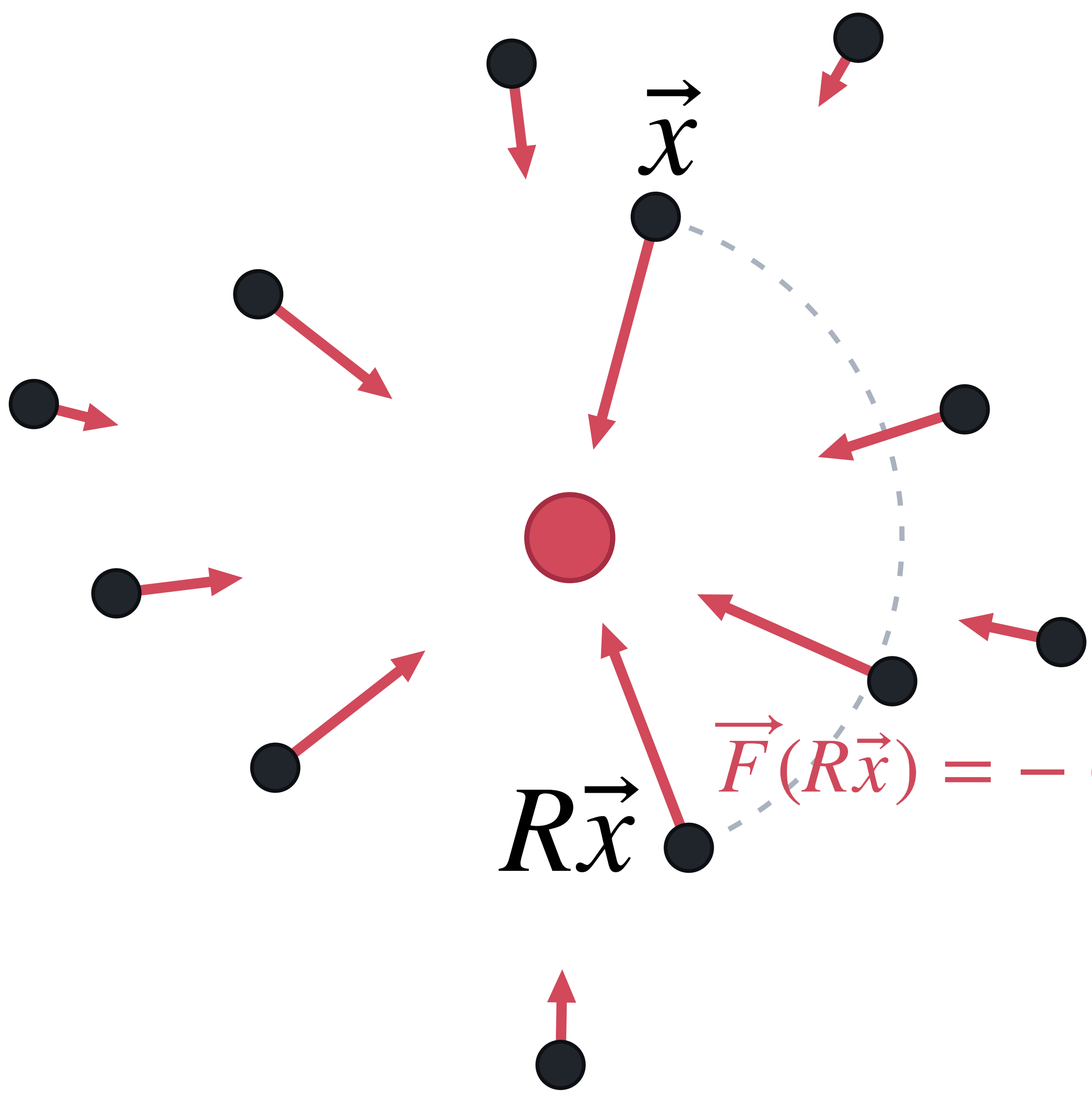
Inductive bias

Example: learn force field between two charged particles



Inductive bias

Example: learn force field between two charged particles



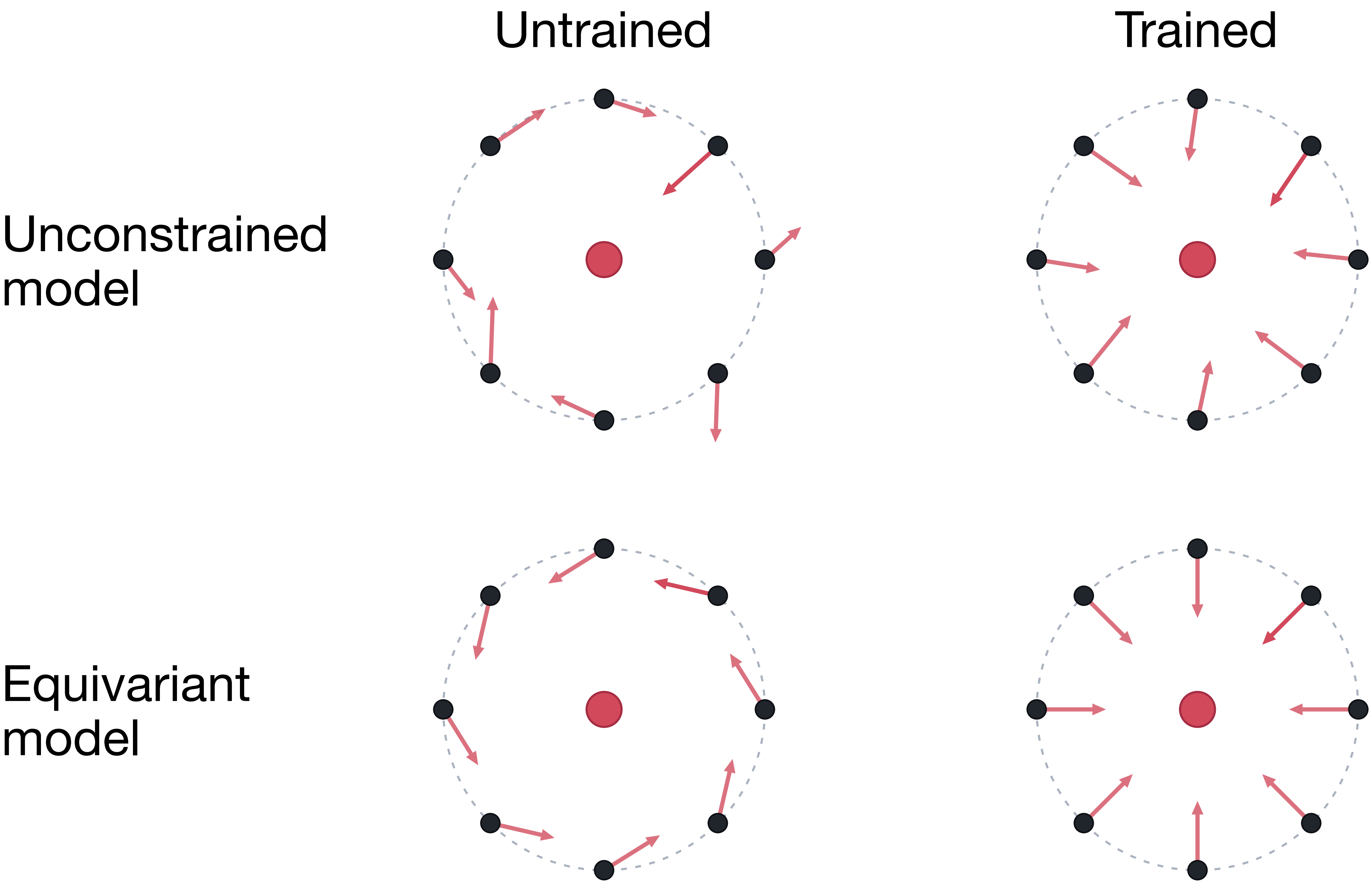
$$\vec{F}(R\vec{x}) = -C\frac{R\vec{x}}{|\vec{x}|^3} = R\vec{F}(\vec{x})$$

Rotational equivariance under

$$R \in \mathrm{SO}(2)$$

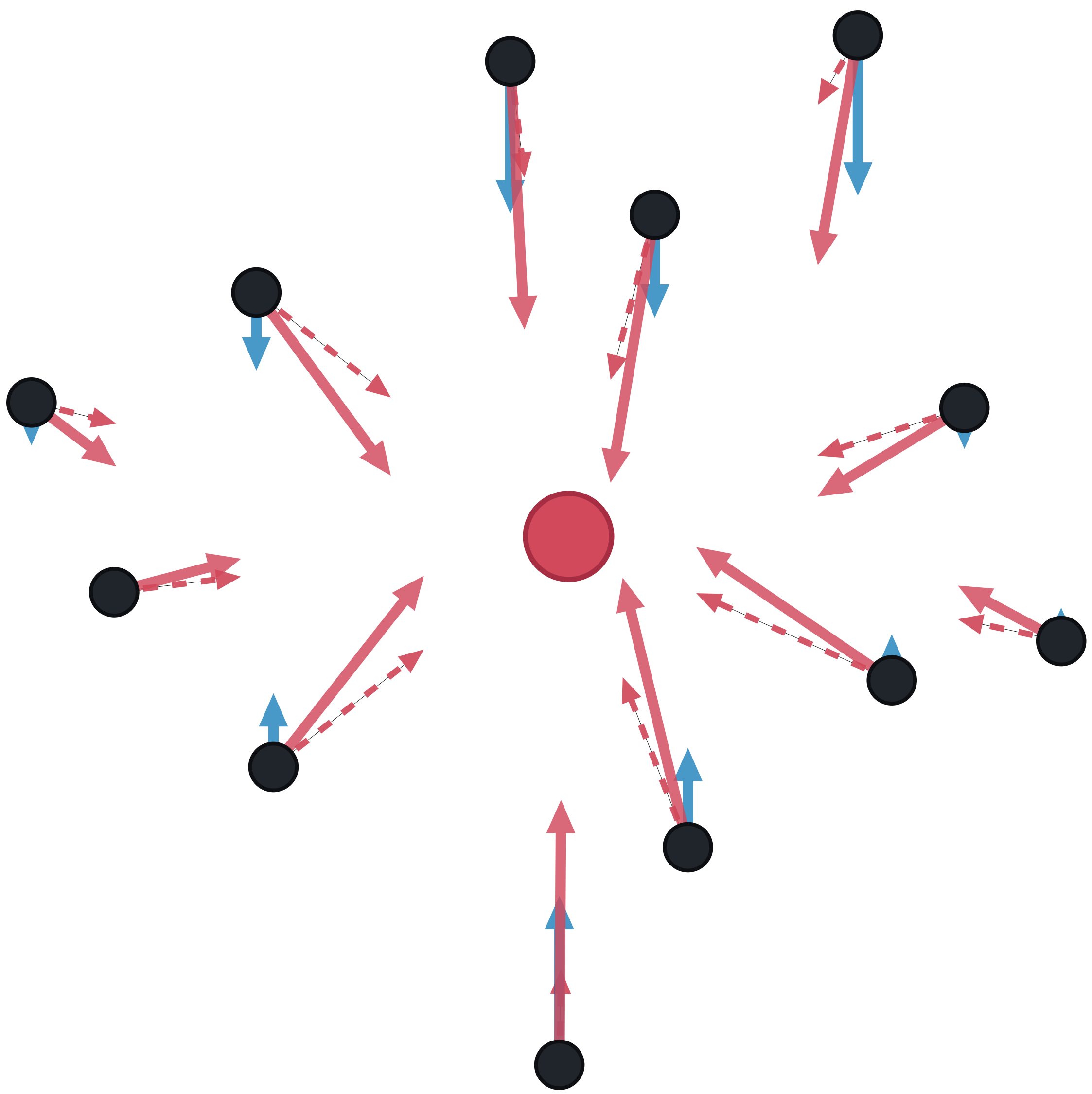
Inductive bias

Equivariance



Broken equivariance

Example: anisotropic potential



Approximate symmetry

Prior work

Boundary conditions

External forces

Noise

Omitted context

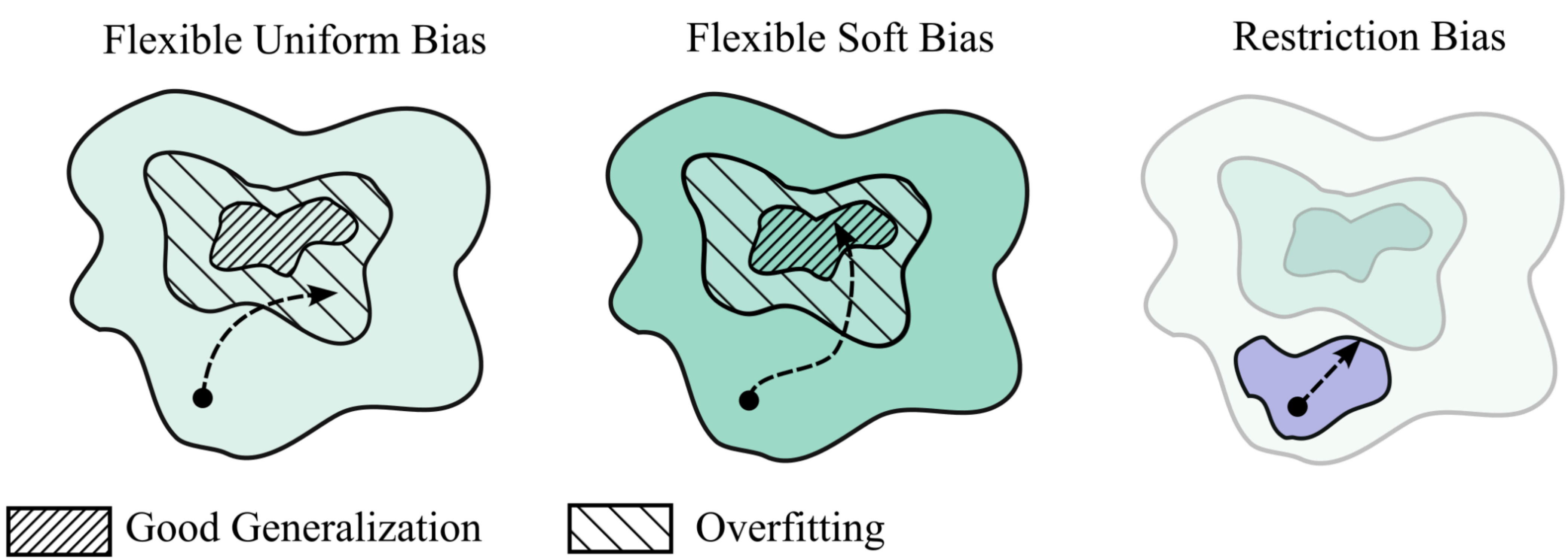
$$f(gx) \neq gf(x)$$

$$f(gx, gc) = gf(x, c)$$

Finzi et al., "Residual Pathway Priors for Soft Equivariance Constraints," **NeurIPS**, 2021.
Wang et al., "Approximately Equivariant Networks for Imperfectly Symmetric Dynamics," **ICML**, 2022.
Romero et al., "Learning Partial Equivariances From Data," **NeurIPS**, 2022.
van der Ouderaa et al., "Relaxing Equivariance Constraints with Non-stationary Continuous Filters," **NeurIPS**, 2022.
Kim et al., "Regularizing Towards Soft Equivariance Under Mixed Symmetries," **ICML**, 2023.
Petrache et al., "Approximation-Generalization Trade-offs under (Approximate) Group Equivariance," **NeurIPS**, 2023.
Huang et al., "Approximately Equivariant Graph Networks," **NeurIPS**, 2023.
Ashman et al., "Approximately Equivariant Neural Processes," **NeurIPS**, 2024

Inductive bias

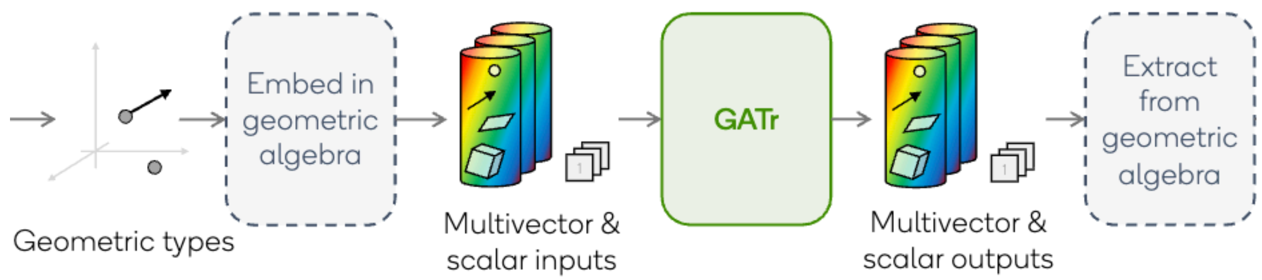
What is the right amount?

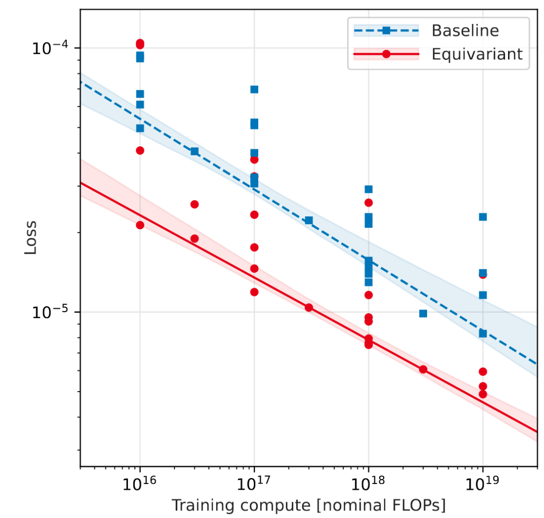


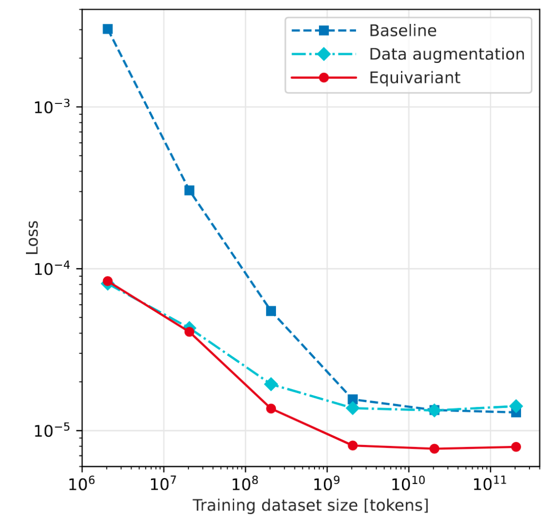
A. G. Wilson, "Deep Learning is Not So Mysterious or Different", arXiv:2503.02113

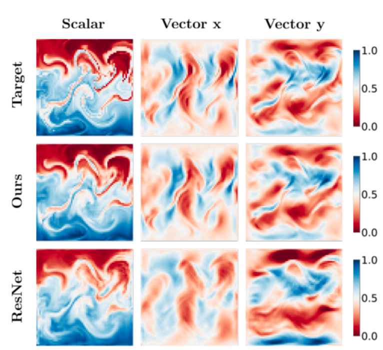
Geometric inductive bias

Clifford algebra neural networks



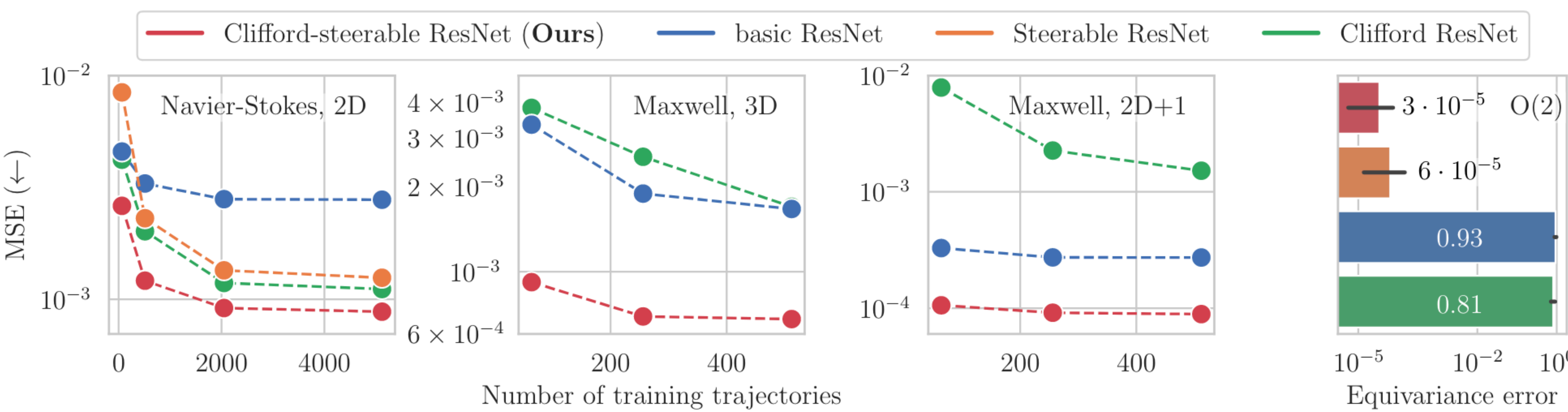






Brehmer, Johann, Sönke Behrends, Pim de Haan, and Taco Cohen. “Does Equivariance Matter at Scale?” **TMLR**, arXiv:2410.23179

Brehmer, Johann, Pim De Haan, Sönke Behrends, and Taco S. Cohen. "Geometric algebra transformer.” **NeurIPS 2023**



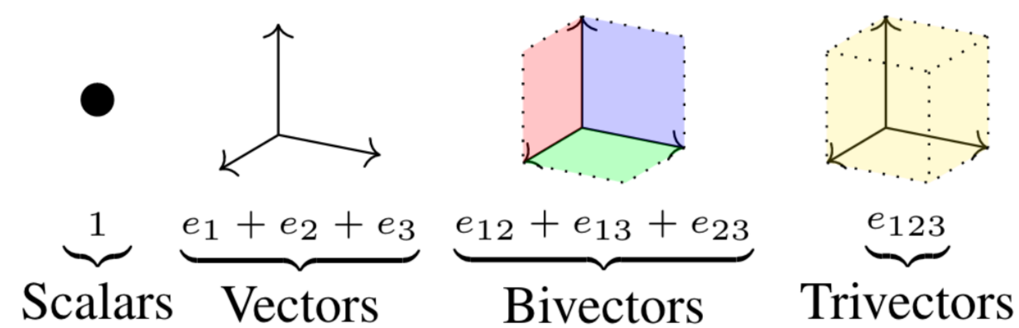
Zhdanov, Maksim, David Ruhe, Maurice Weiler, Ana Lucic, Johannes Brandstetter, and Patrick Forré. "Clifford-steerable convolutional neural networks.” **ICML 2024**

Geometric inductive bias

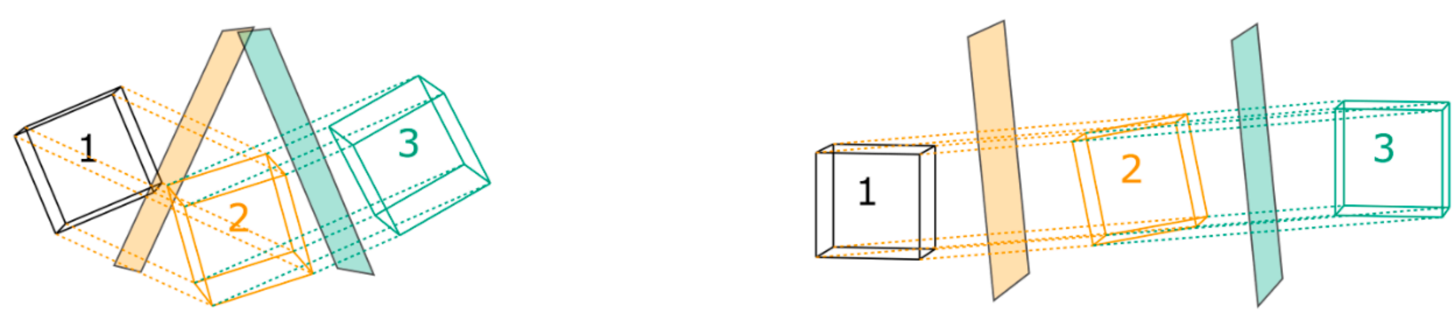
Clifford algebra / Geometric algebra

$$\mathbf{e}_i\mathbf{e}_i \in \{+1, -1, 0\}$$

$$\mathbf{e}_i\mathbf{e}_j = -\mathbf{e}_j\mathbf{e}_i \quad (i \neq j)$$



$$x = x_s + x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3 + x_{12}\mathbf{e}_1\mathbf{e}_2 + x_{13}\mathbf{e}_1\mathbf{e}_3 + x_{23}\mathbf{e}_2\mathbf{e}_3 + x_{123}\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3$$



Ruhe, David, Jayesh K. Gupta, Steven De Keninck, Max Welling, and Johannes Brandstetter. "Geometric clifford algebra networks." ICML, 2023.

Brehmer, Johann, Pim de Haan, Sönke Behrends, and Taco Cohen. "Geometric Algebra Transformer," NeurIPS 2023

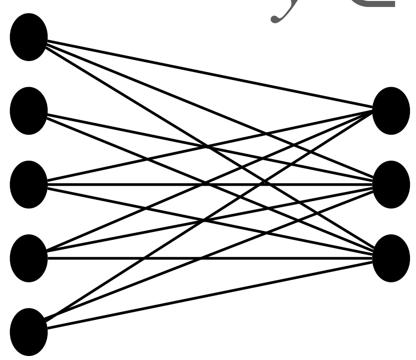
Geometric inductive bias

Clifford algebra neural networks

MLP

$x \in \mathbb{R}^m$

$y \in \mathbb{R}^n$

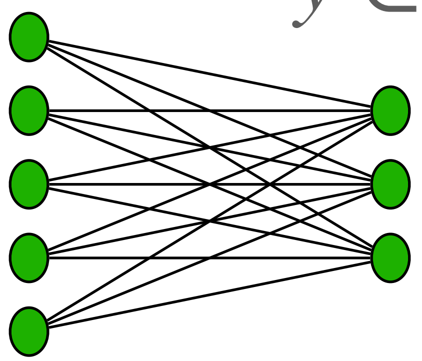


$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Clifford MLP

$x \in \mathbb{R}^m \times \mathbb{R}^8$

$y \in \mathbb{R}^n \times \mathbb{R}^8$



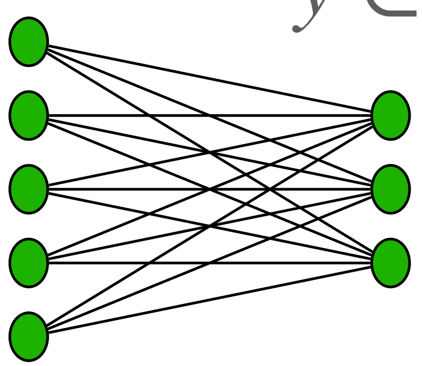
$$\begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \cdots & \mathbf{a}_{1m} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \cdots & \mathbf{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_{n1} & \mathbf{a}_{n2} & \cdots & \mathbf{a}_{nm} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_m \end{bmatrix}$$

Geometric inductive bias

Clifford algebra: implementation

$x \in \mathbb{R}^m \times \mathbb{R}^8$

$y \in \mathbb{R}^n \times \mathbb{R}^8$

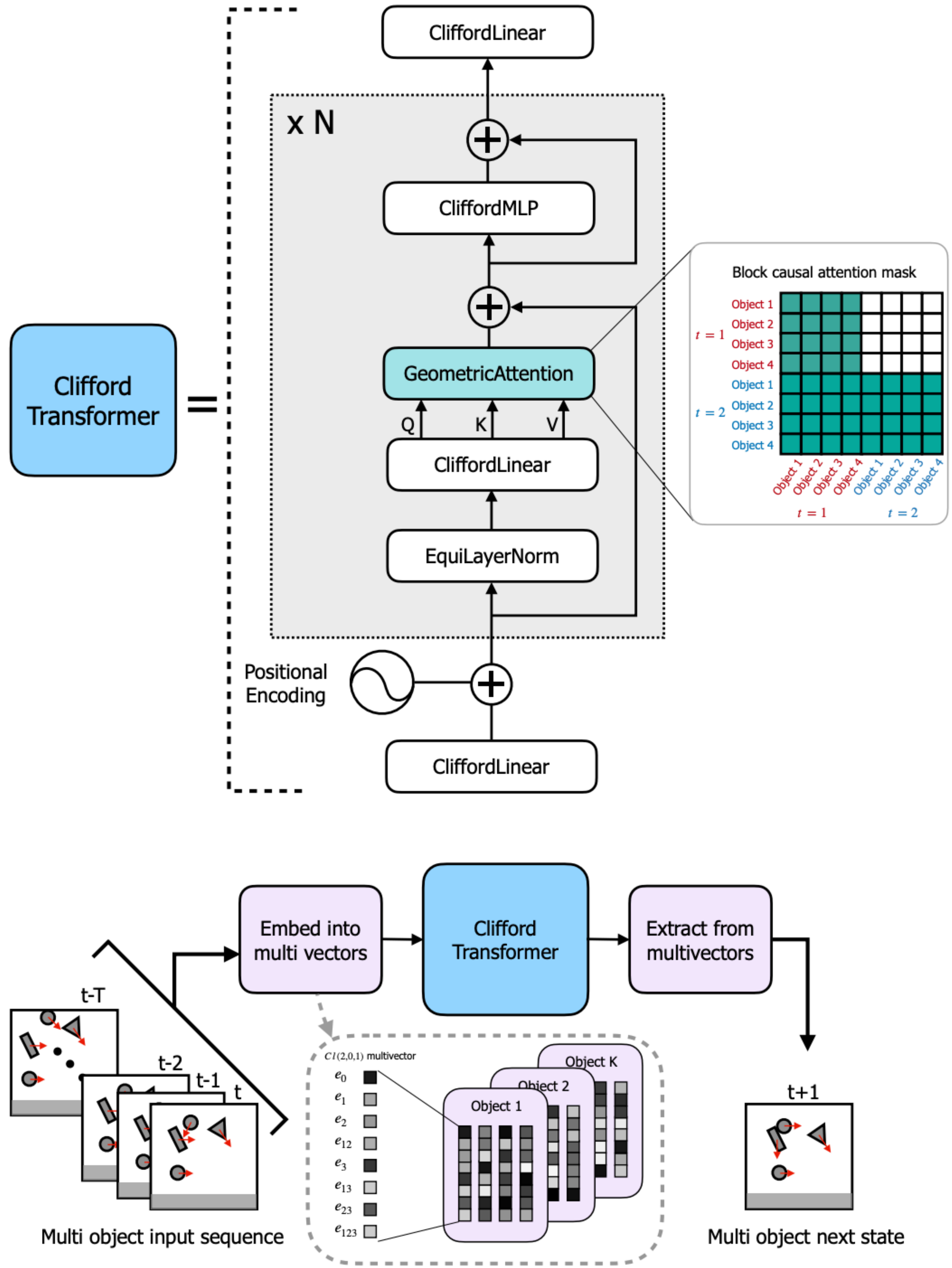


$$\begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \cdots & \mathbf{a}_{1m} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \cdots & \mathbf{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_{n1} & \mathbf{a}_{n2} & \cdots & \mathbf{a}_{nm} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_m \end{bmatrix}$$

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

$$\begin{aligned} \mathbf{y}^i(x) = \mathbf{a}^{ij}\mathbf{x}_j &= (a^{ij\alpha}\mathbf{e}_\alpha)(x_j{}^\beta \mathbf{e}_\beta) \\ &= a^{ij\alpha} x_j{}^\beta\, \mathbf{e}_\alpha \mathbf{e}_\beta \\ &= a^{ij\alpha} x_j{}^\beta f_{\alpha\beta\gamma}\, \mathbf{e}^\gamma \end{aligned}$$

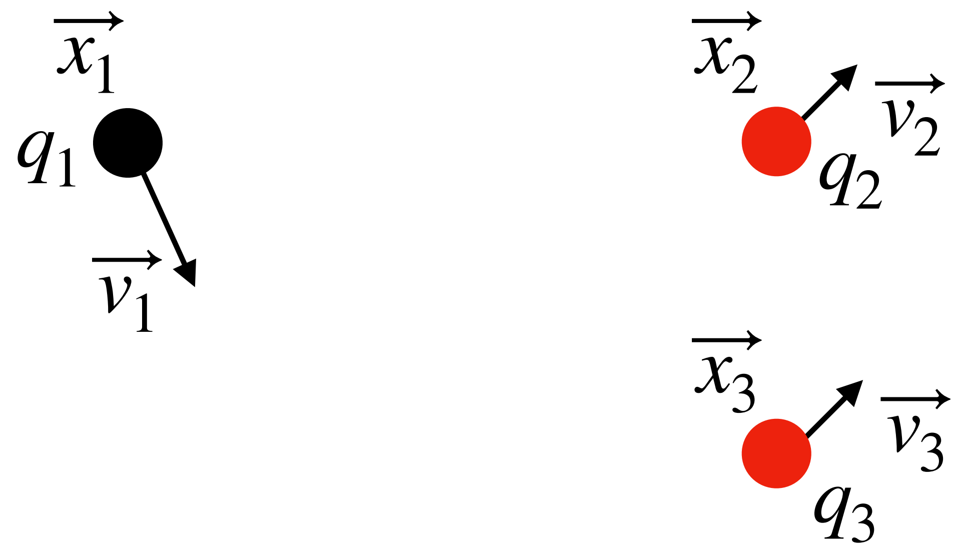
```
                  [m, n, 8]   [8, 8, 8]
einsum("ijk,kj,klm", W, x, f)
                          [m, 8]
```



CliffordLinear
x N
CliffordMLP
GeometricAttention
Q
K
V
CliffordLinear
EquiLayerNorm
Positional Encoding
CliffordLinear
Clifford Transformer
=
Block causal attention mask
Object 1
Object 2
Object 3
Object 4
t = 1
t = 2
Embed into multi vectors
Clifford Transformer
Extract from multivectors
t-T
t-2
t-1
t
t+1
Cl(2,0,1) multivector
e0
e1
e2
e12
e3
e13
e23
e123
Object 1
Object 2
Object K
Multi object input sequence
Multi object next state

Object centric dynamics

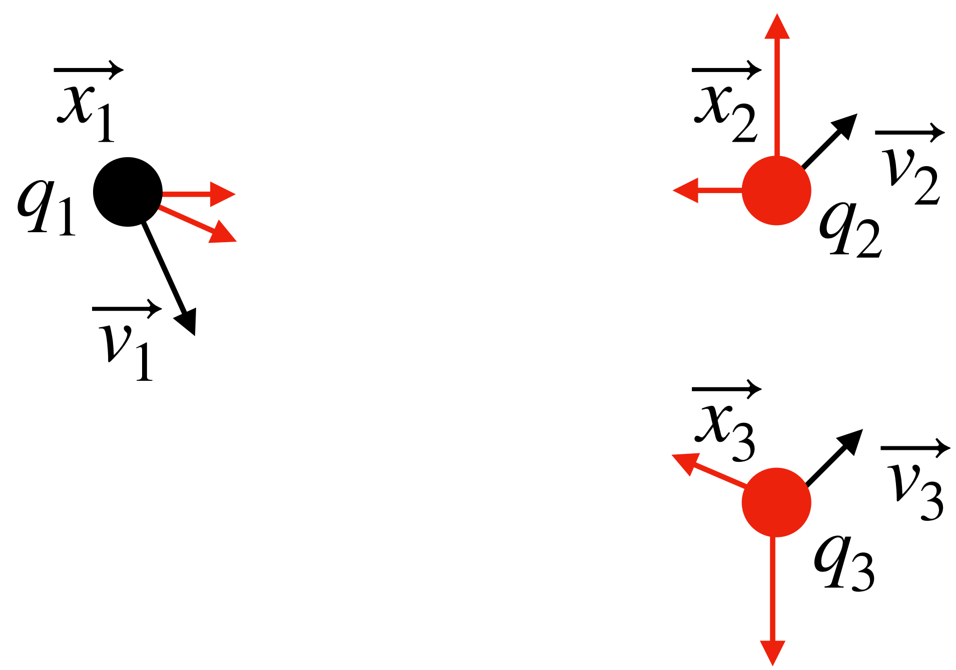
charged particles



Object centric dynamics

charged particles

State



Dynamics

$$\overrightarrow{F}_i = \sum_{j \neq i} q_i q_j \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|^2}$$

$$\frac{d\vec{v}_i}{dt} = \overrightarrow{F}_i$$

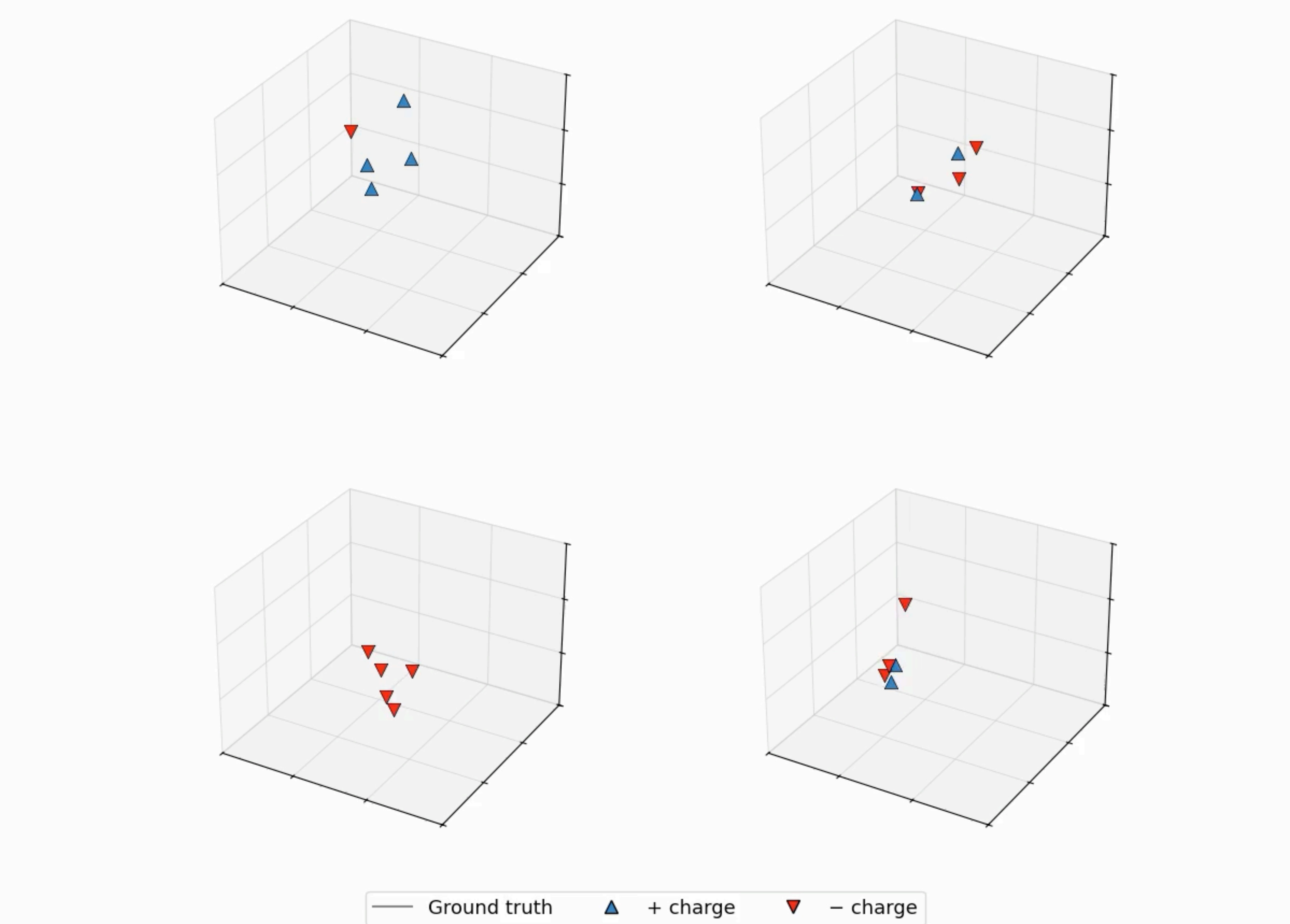
Task

predict $\vec{x}_i(t + \Delta t)$ $\vec{v}_i(t + \Delta t)$

given $\vec{x}_i(t)$ $\vec{v}_i(t)$ q_i

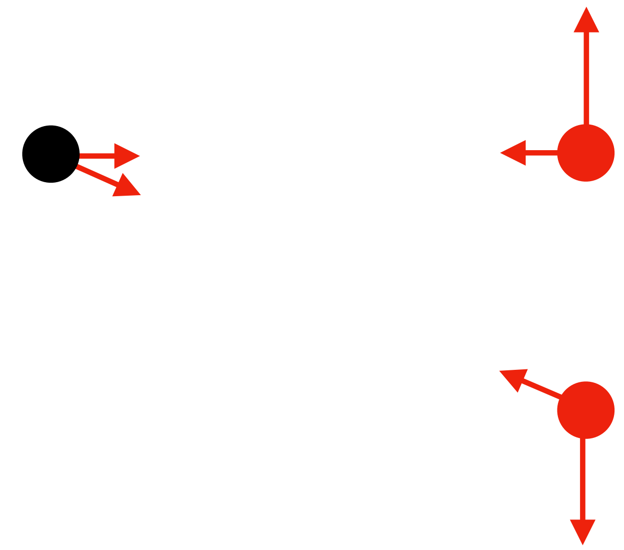
Object centric dynamics

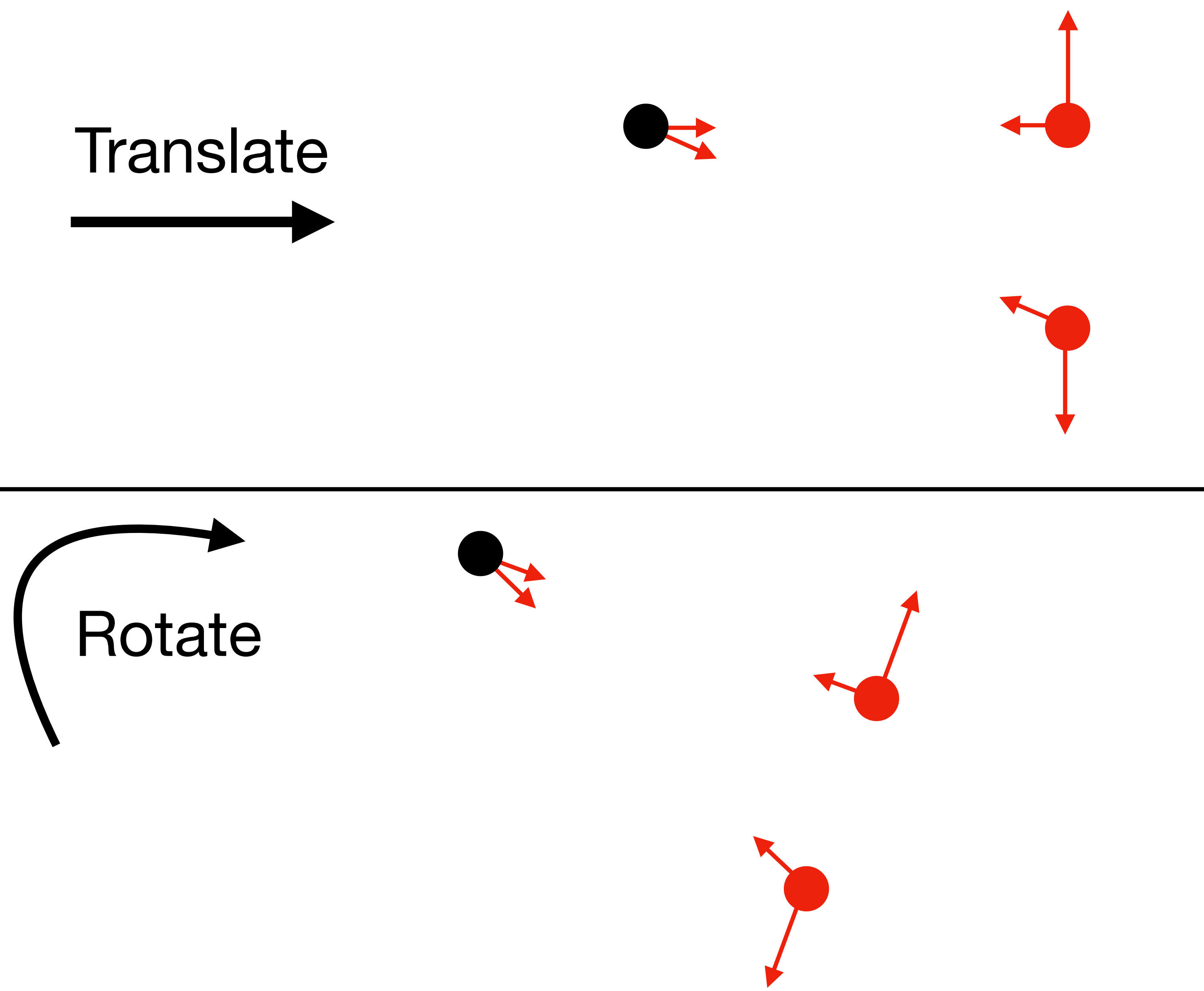
charged particles



Equivariance

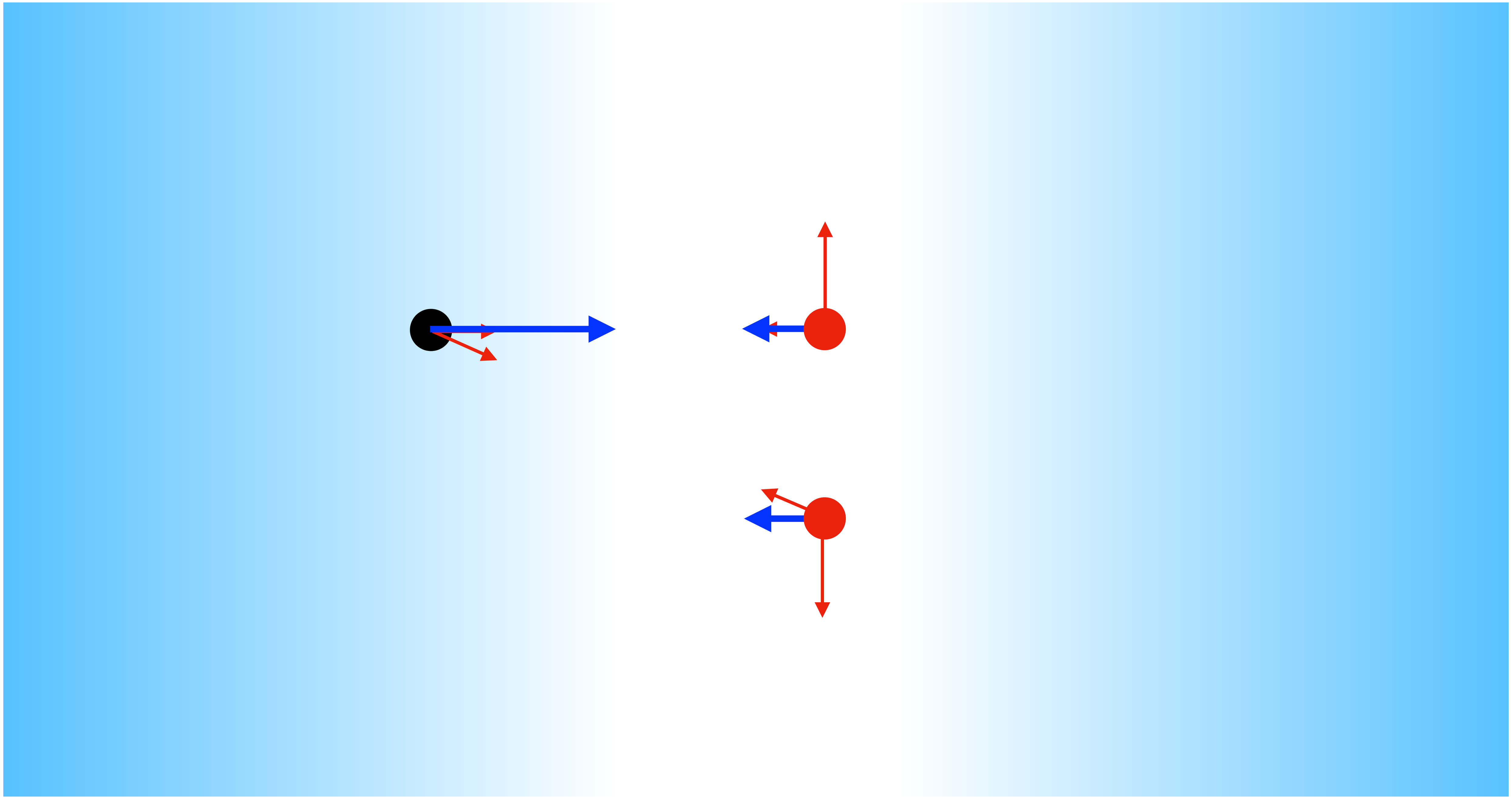
Euclidean group





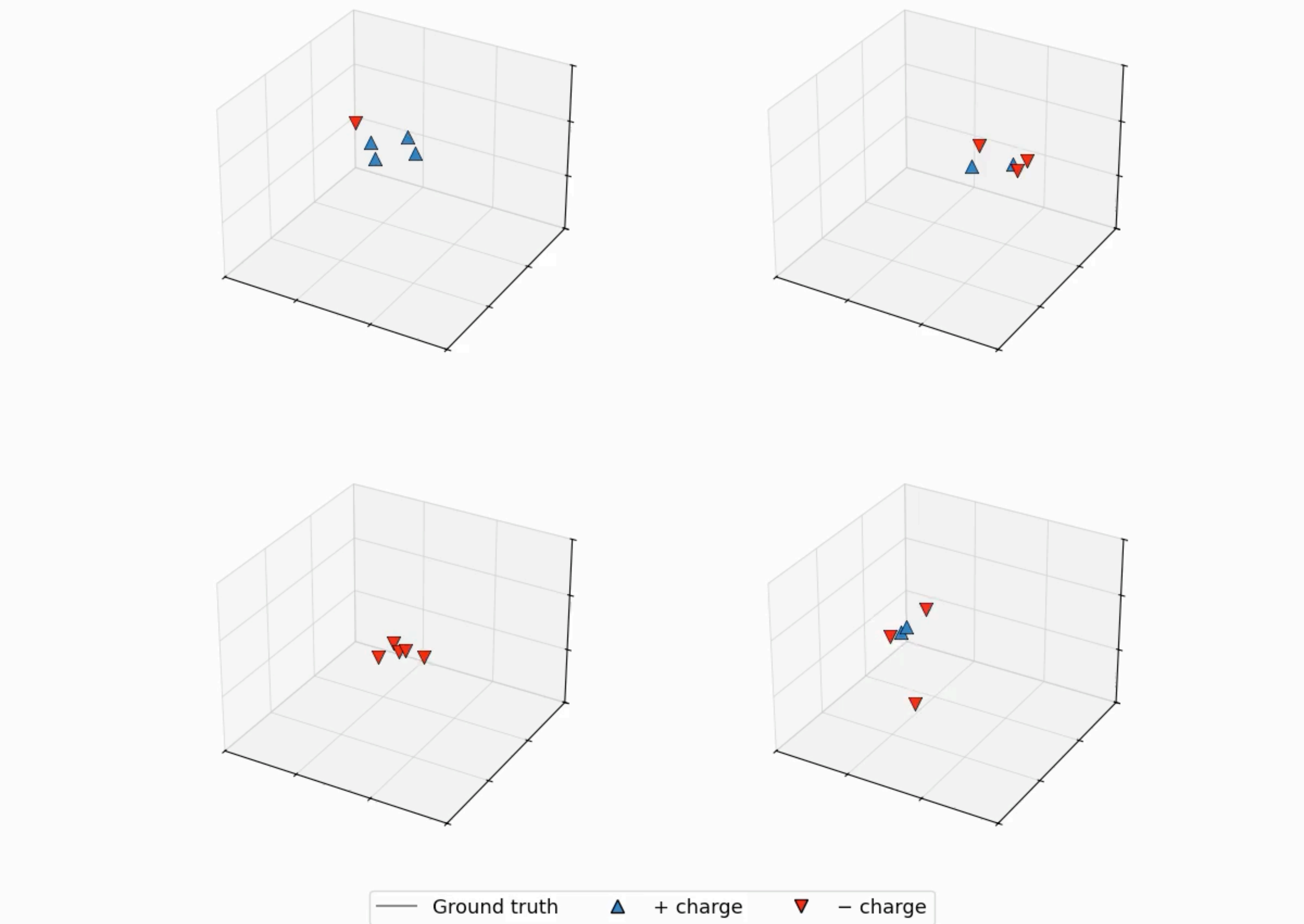
Broken equivariance

charged particles



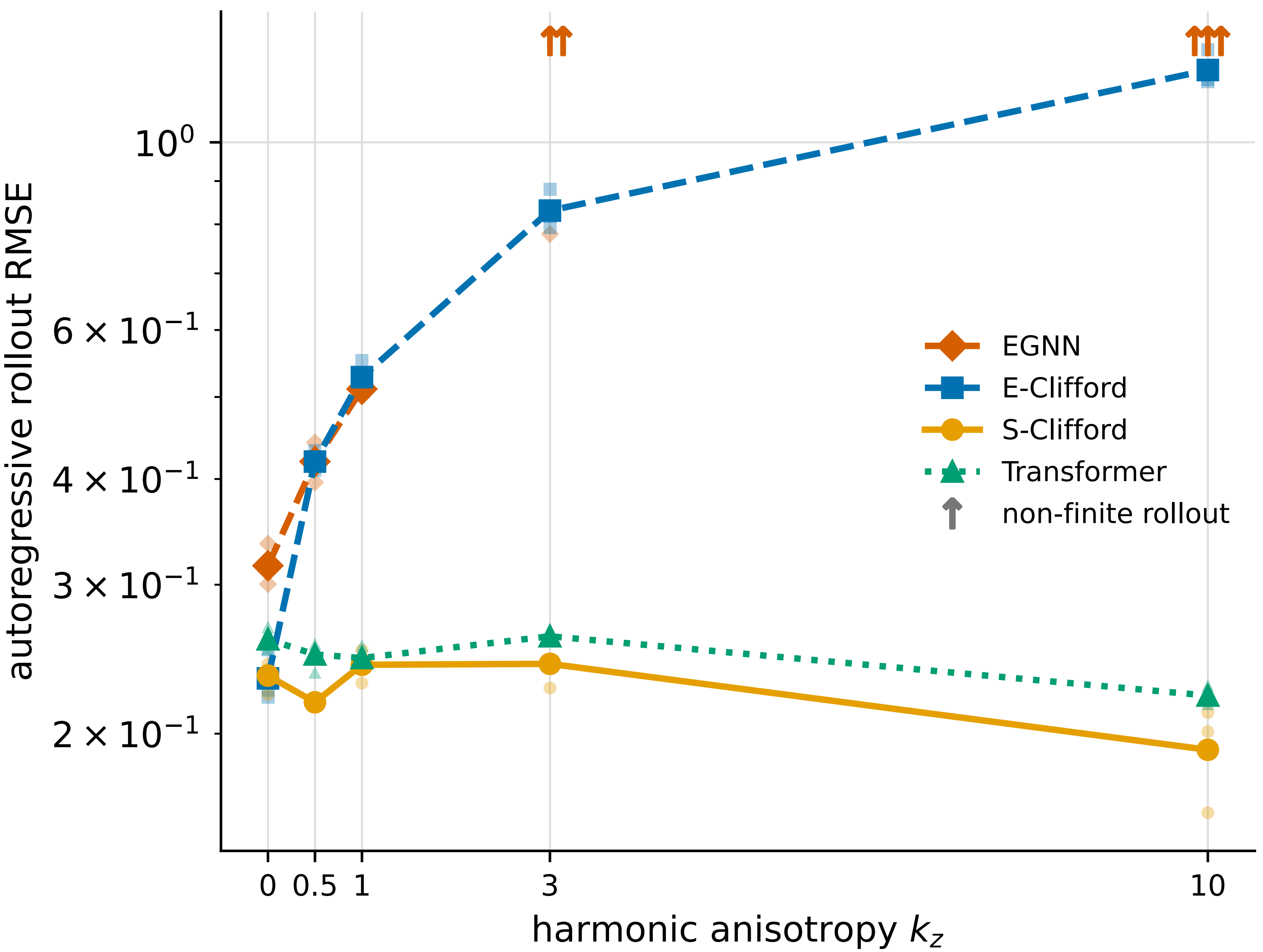
Broken equivariance

charged particles



Results

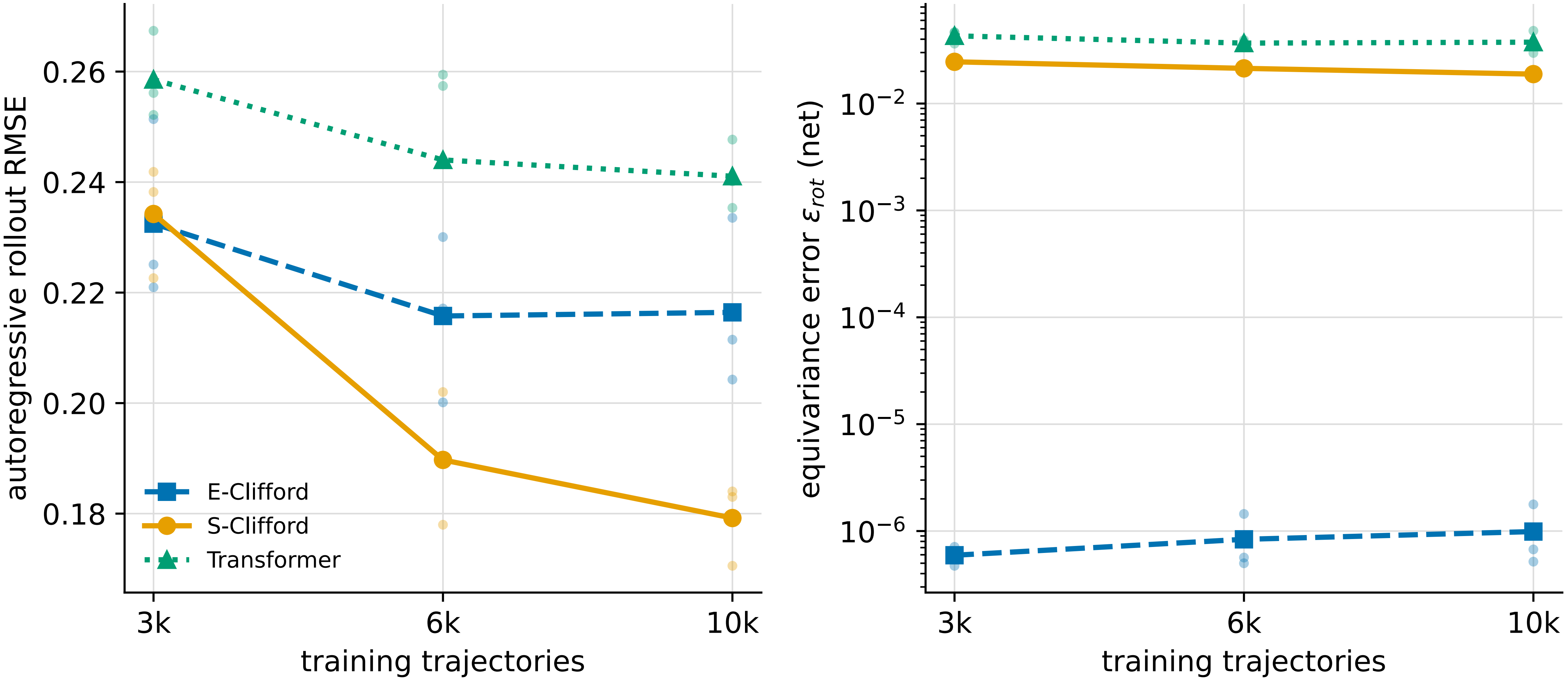
charged particles with broken equivariance



EGNN from **Satorras**, "E (n) equivariant graph neural networks", **PMLR 2021**

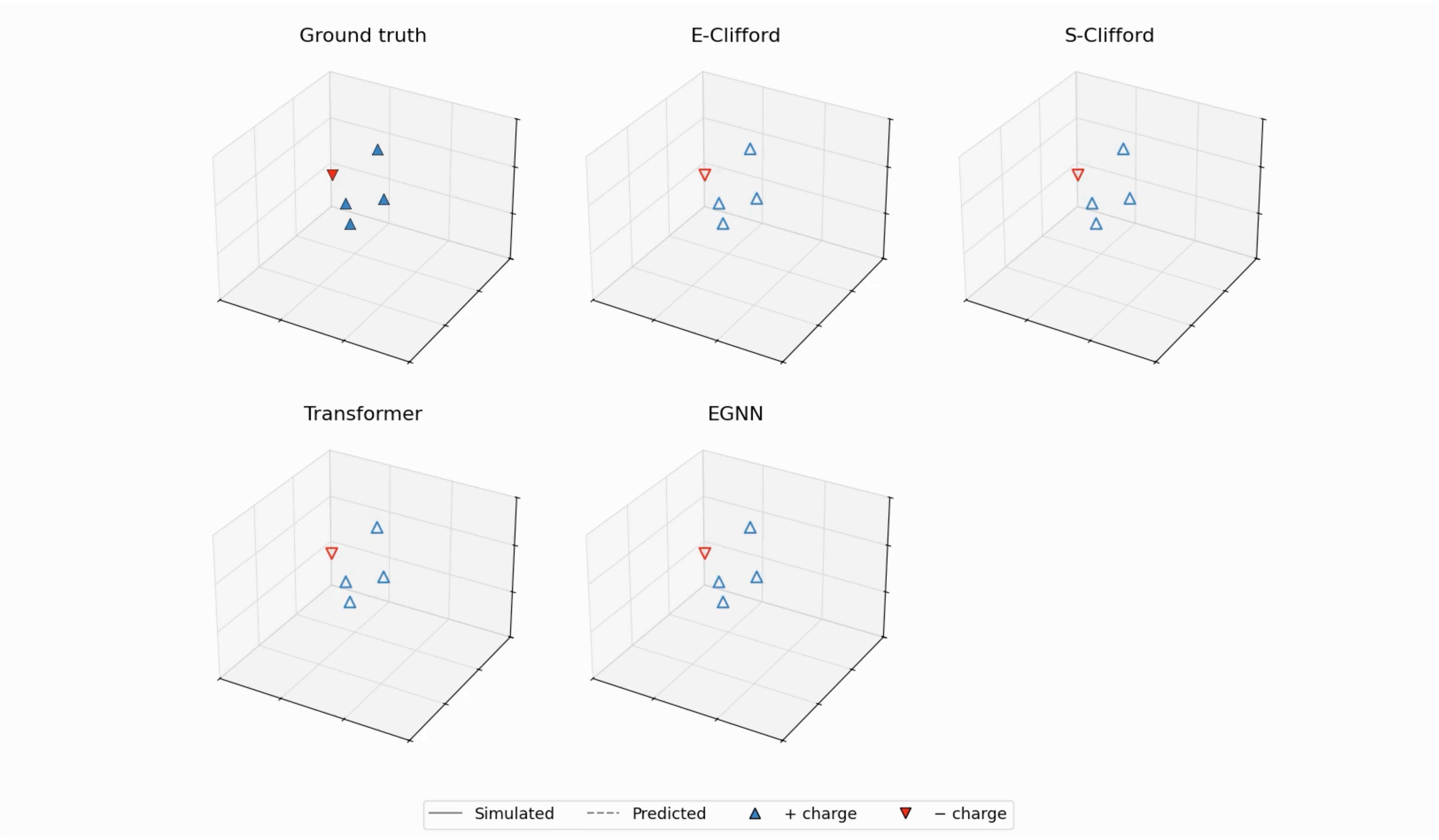
Results

charged particles: data scale with unbroken symmetry



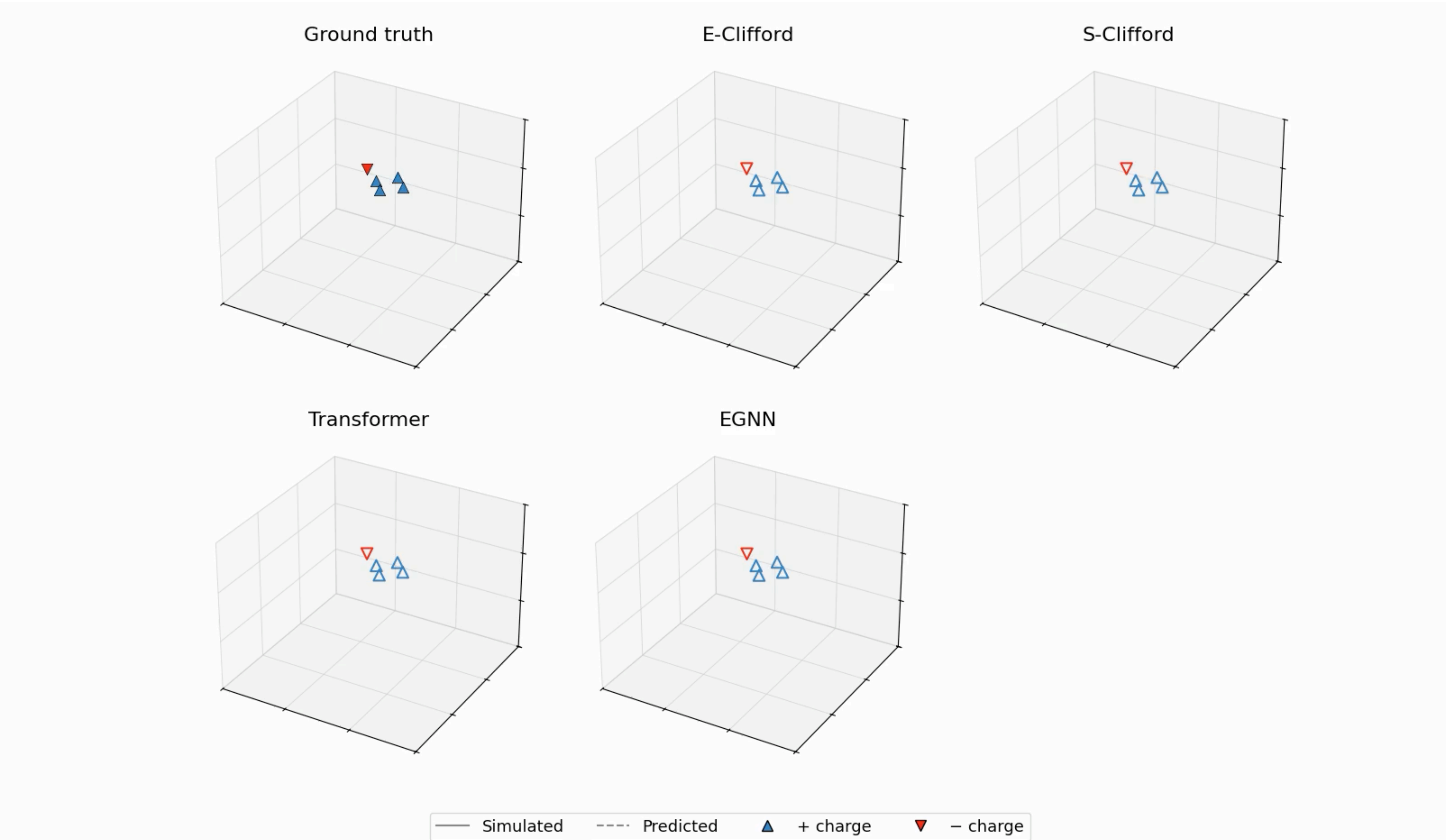
Results

charged particles: rollouts with unbroken symmetry



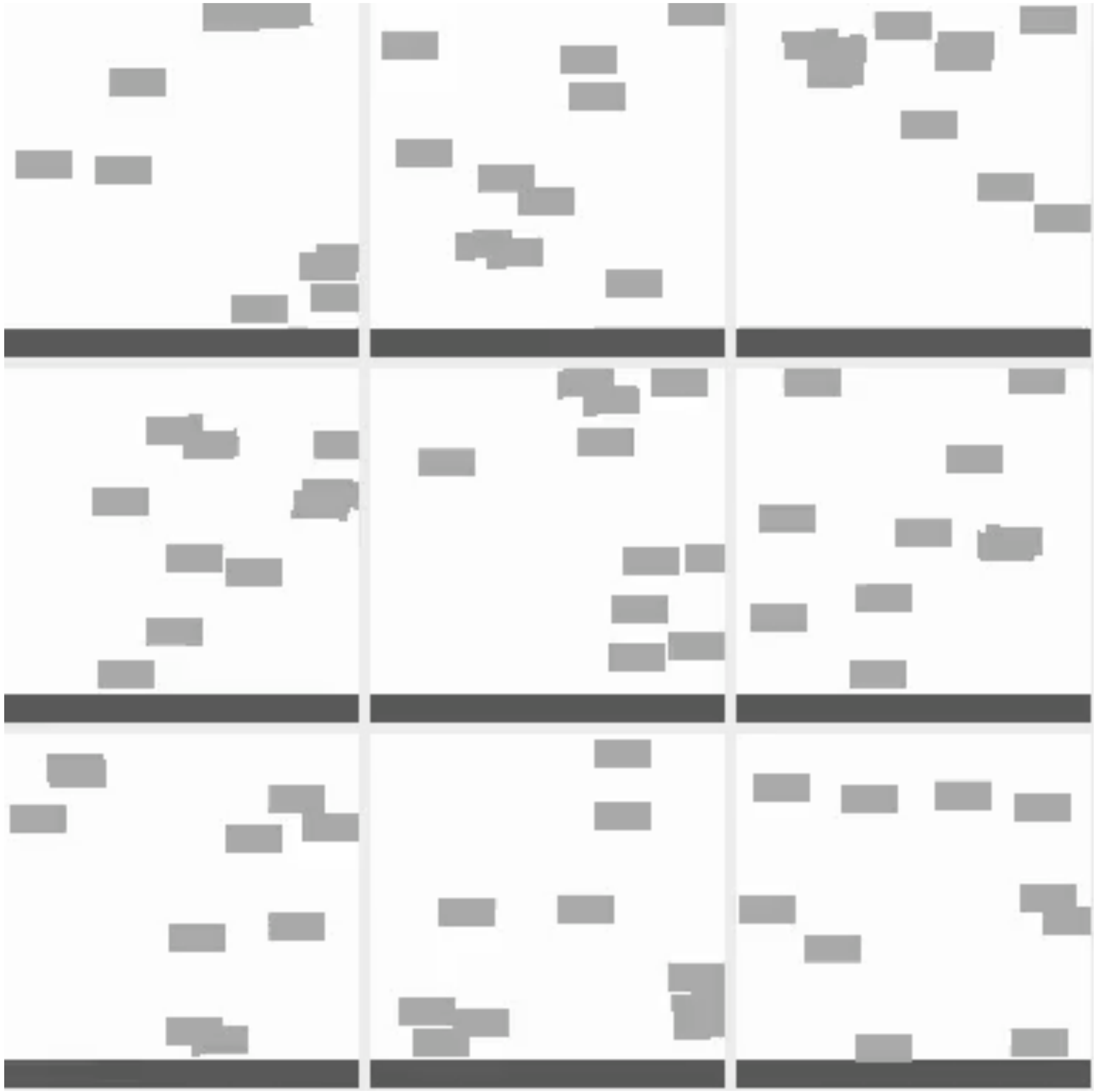
Results

charged particles: rollouts with broken symmetry



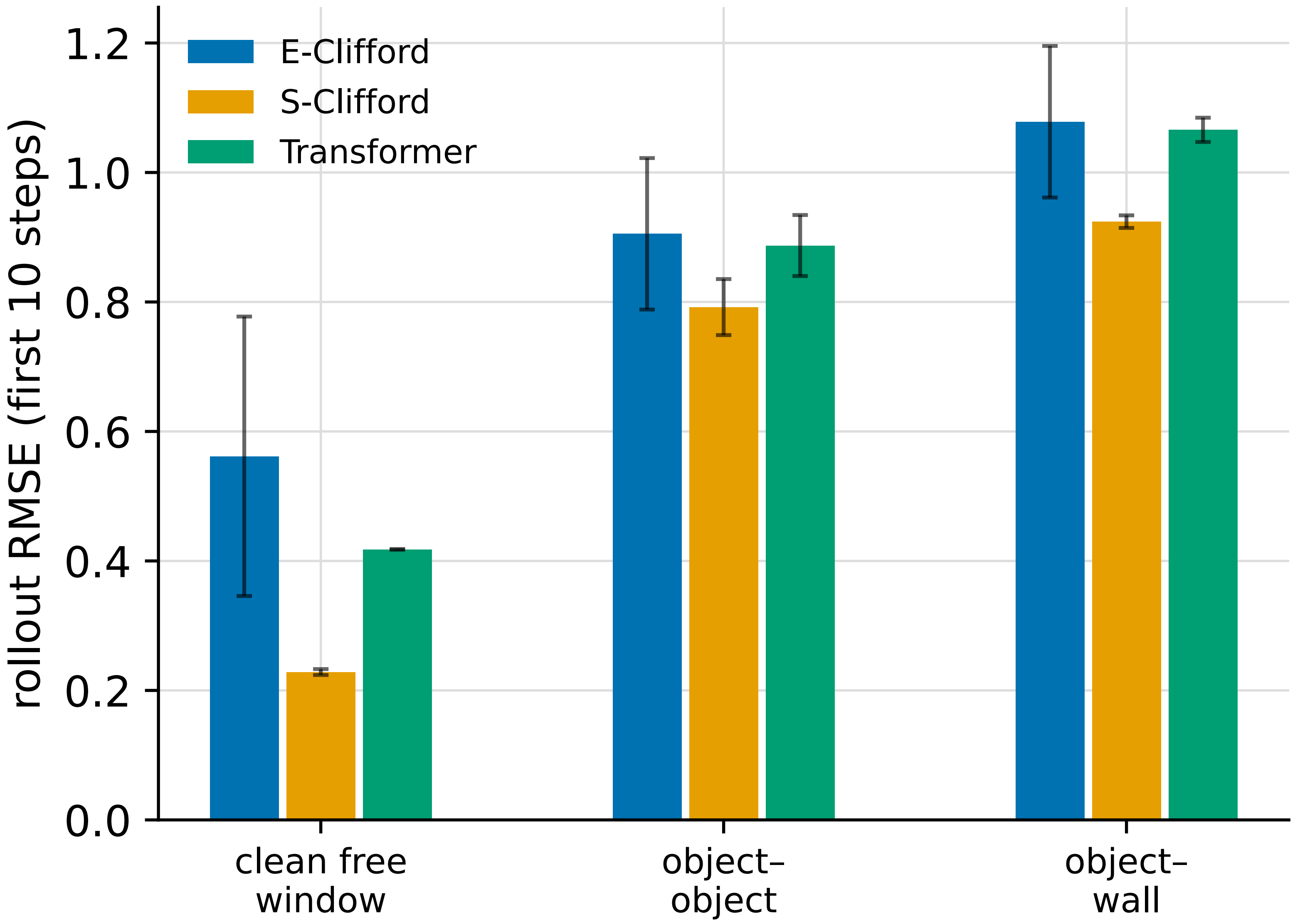
Results

2d rigid body dynamics (Kinetix)



Results

2d rigid body dynamics (Kinetix)



Results

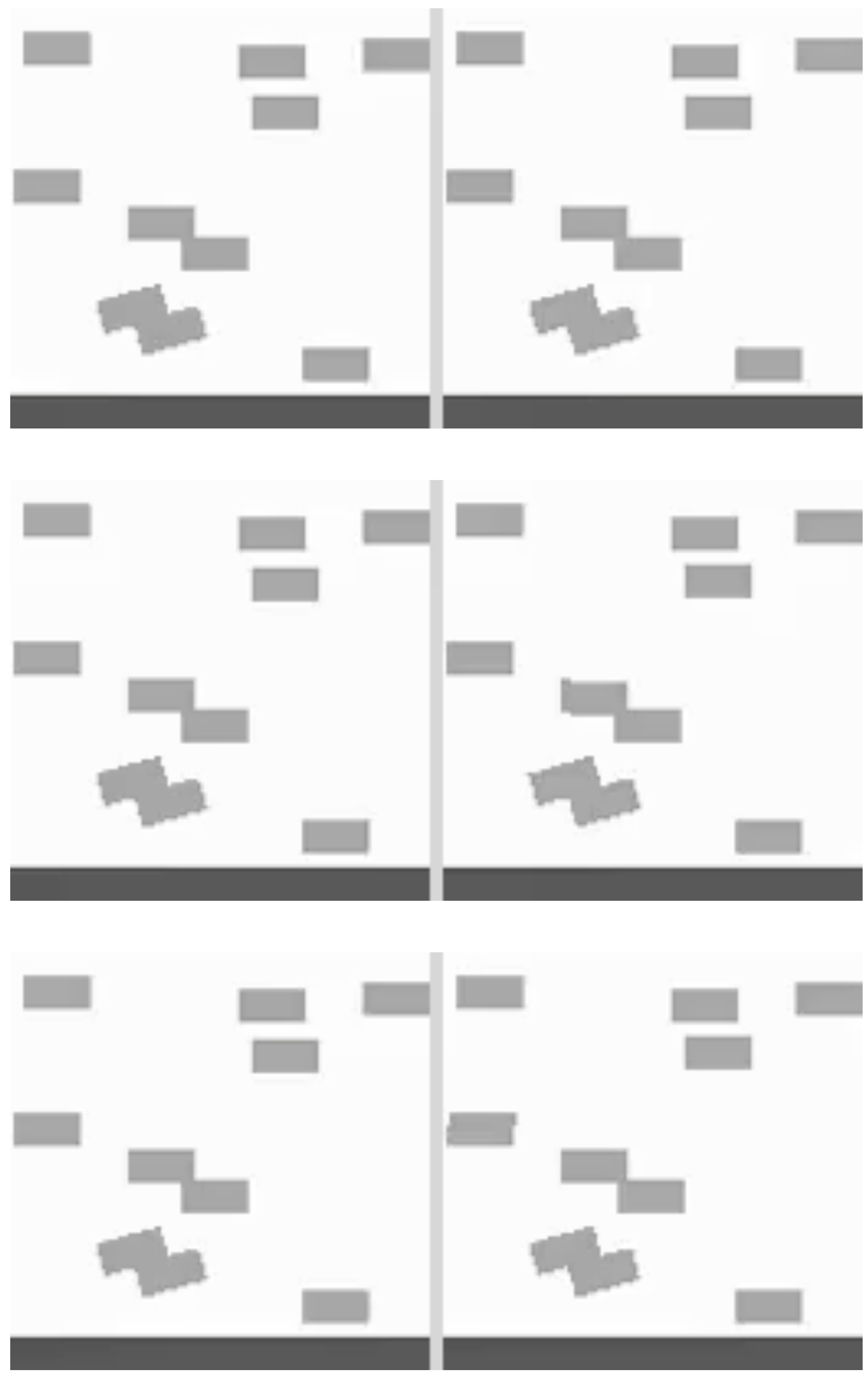
2d rigid body dynamics (Kinetix)

Prediction Simulation

E-Clifford

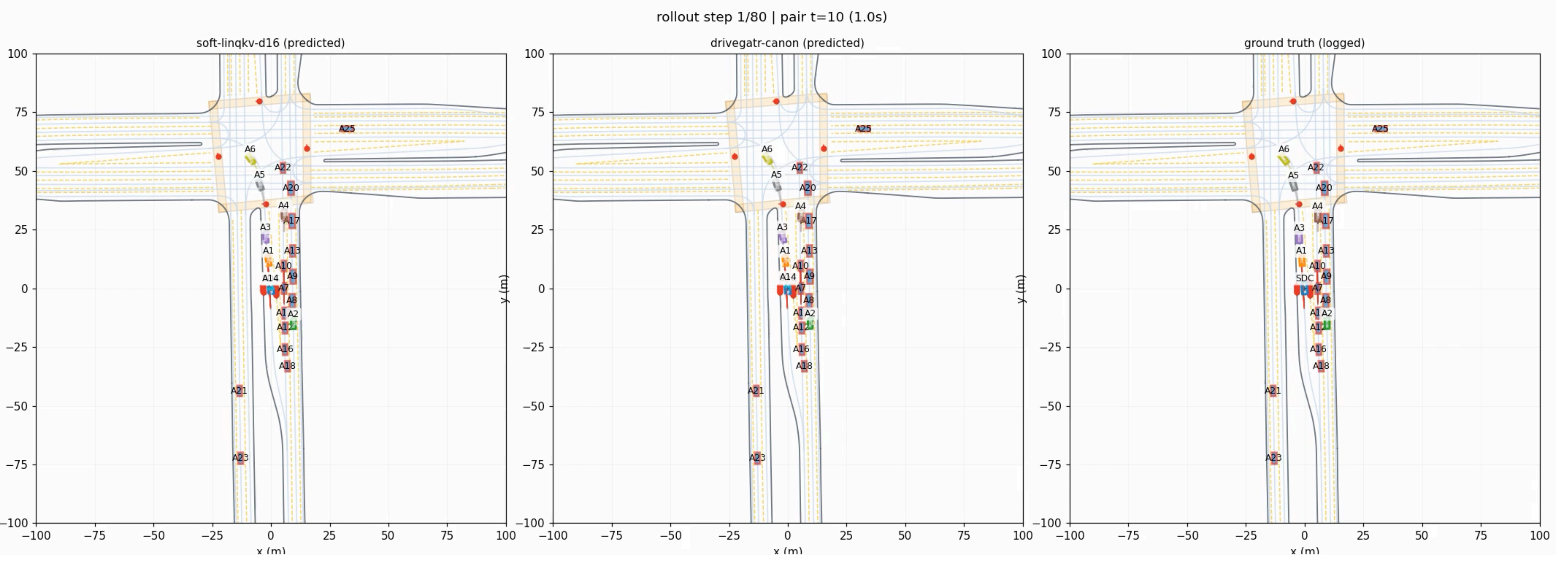
S-Clifford

Transformer



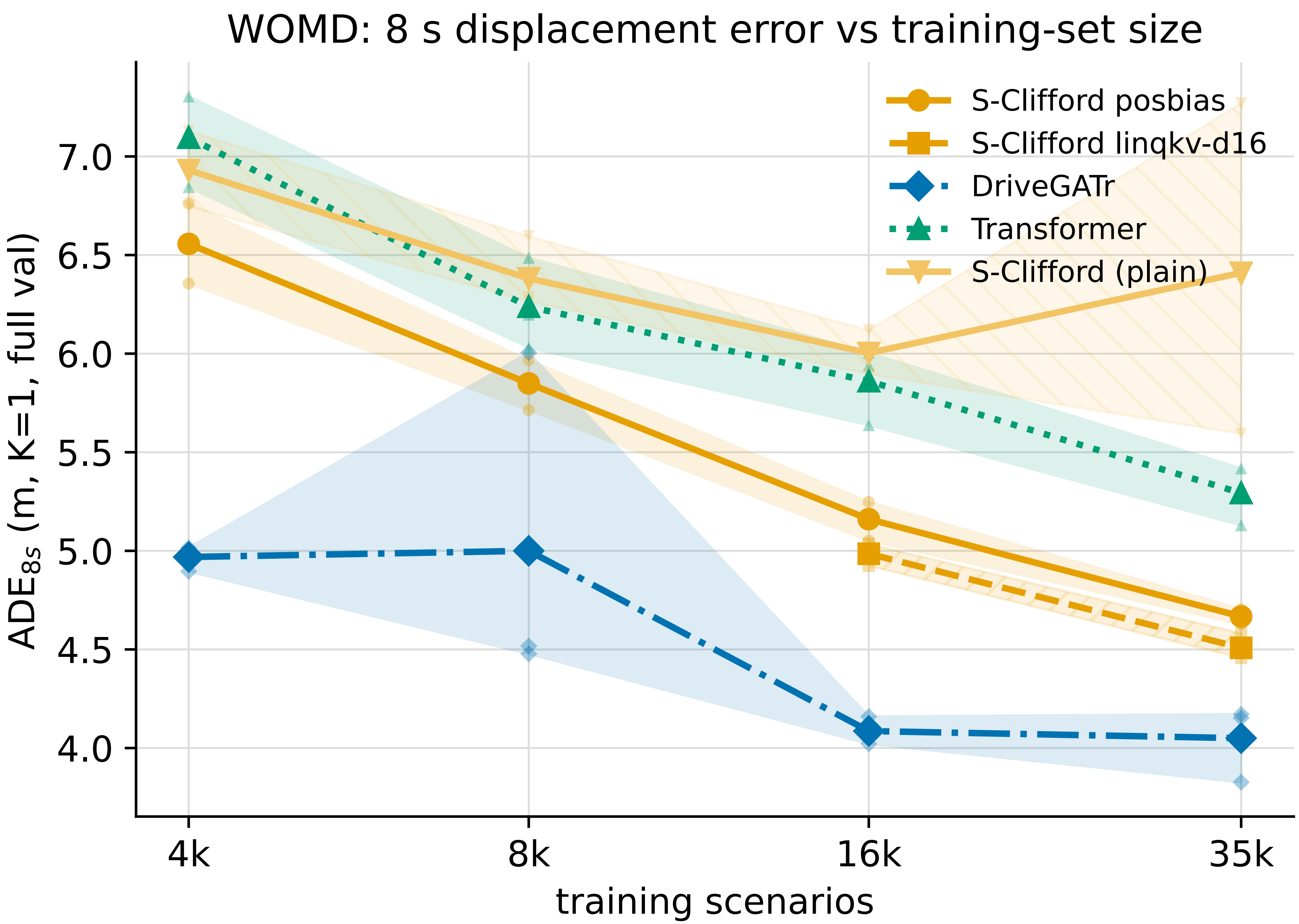
Results

2d multi-agent trajectory prediction (WOMD)

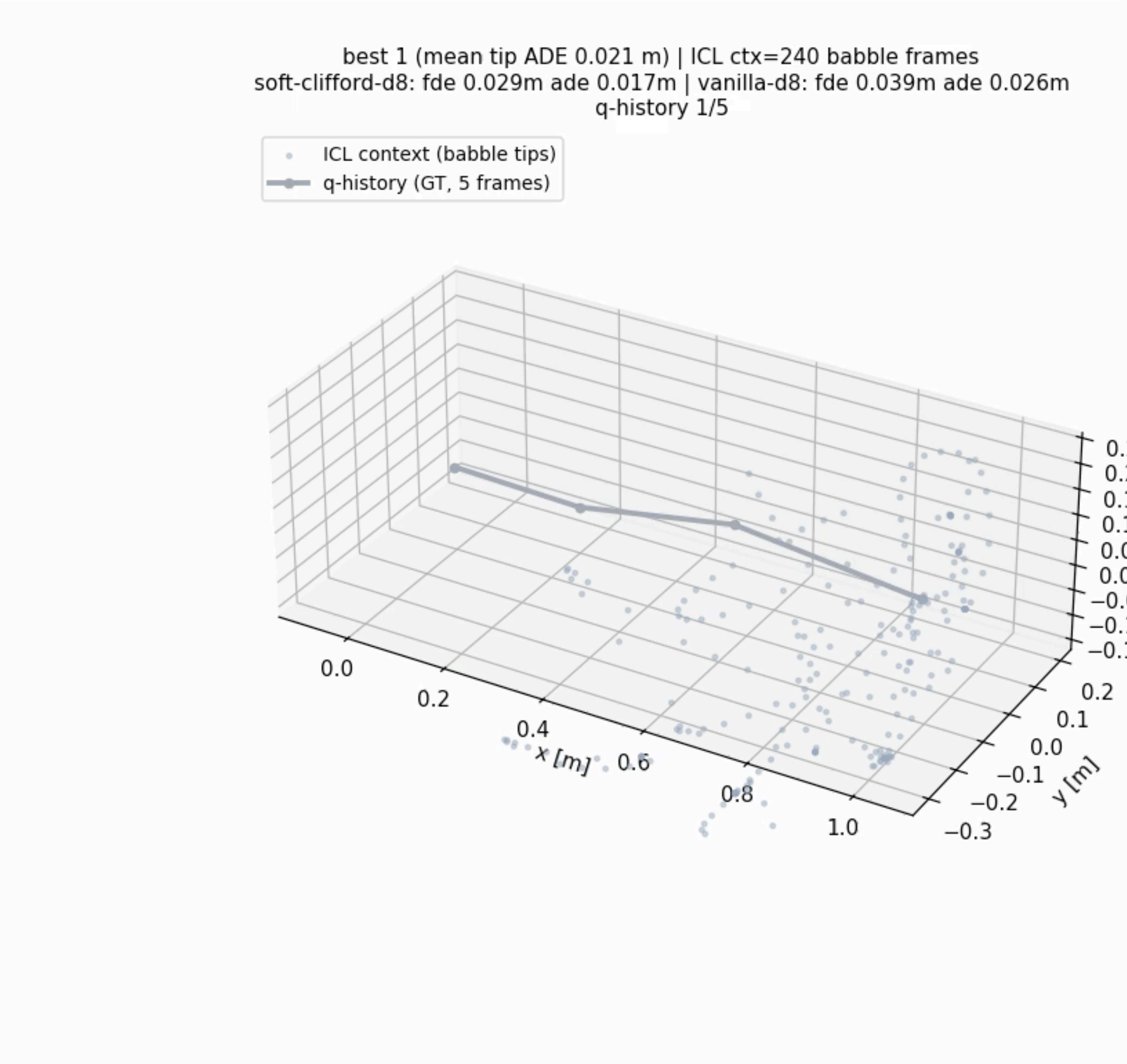


Results

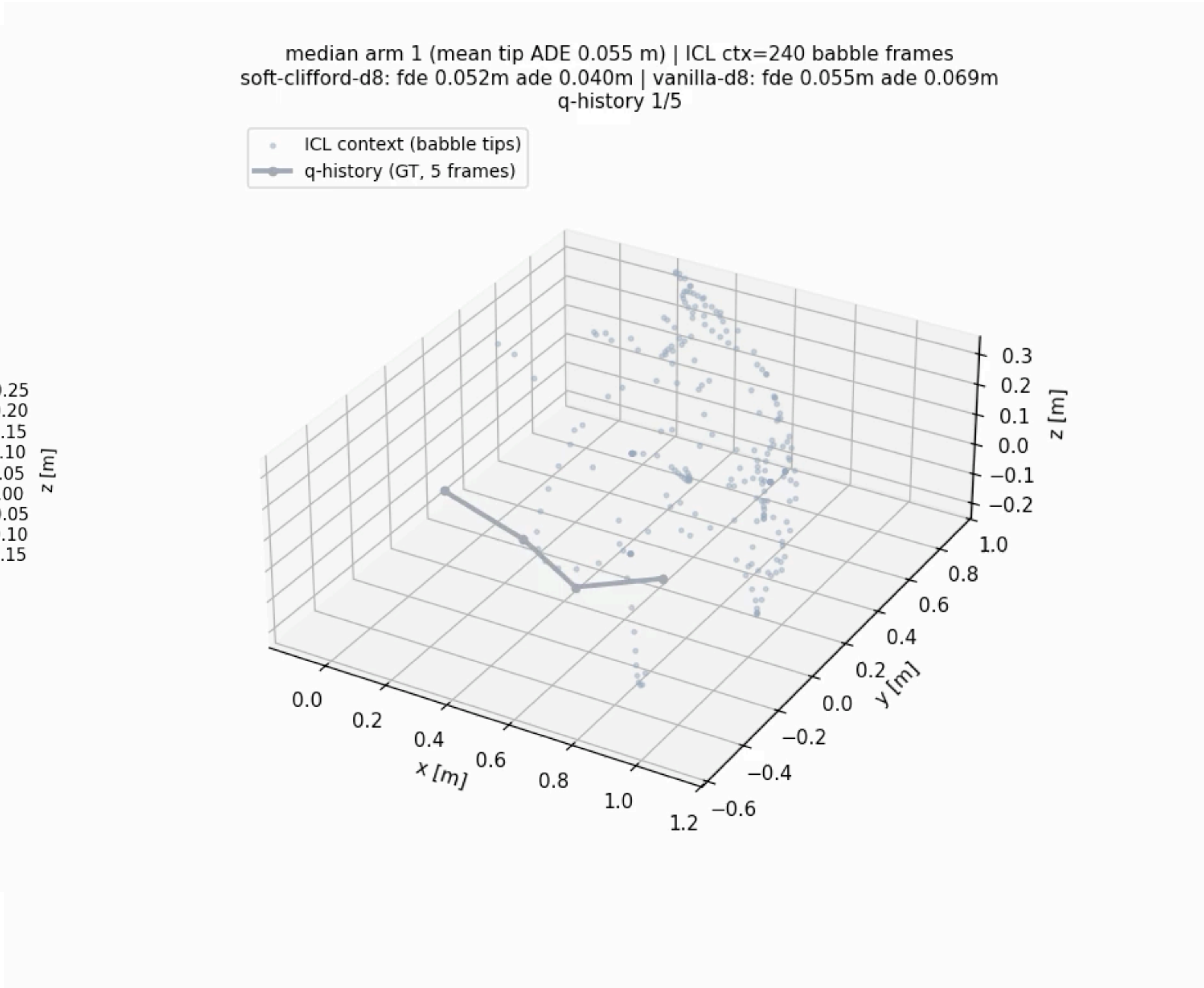
2d multi-agent trajectory prediction (WOMD)



Outlook



best 1 (mean tip ADE 0.021 m) | ICL ctx=240 babble frames
soft-clifford-d8: fde 0.029m ade 0.017m | vanilla-d8: fde 0.039m ade 0.026m
q-history 1/5
ICL context (babble tips)
q-history (GT, 5 frames)
x [m]
y [m]
z [m]



median arm 1 (mean tip ADE 0.055 m) | ICL ctx=240 babble frames
soft-clifford-d8: fde 0.052m ade 0.040m | vanilla-d8: fde 0.055m ade 0.069m
q-history 1/5
ICL context (babble tips)
q-history (GT, 5 frames)
x [m]
y [m]
z [m]

hampus.linander@pm.me

gapindnns.github.io